Global safety formats in *fib* Model Code 2010 for design of concrete structures

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Abstract: Model Code 2010 introduces non-linear analysis in design of concrete structures requires an alternative approach to safety verification and related global safety format. Several methods for verification of limit states using non-linear analysis are presented: full probabilistic method, method ECOV based on the estimate of resistance variation, global safety factor according to EN1992-2 and partial safety factors. The methods are compared on several examples of reinforced concrete structures ranging from ductile to brittle modes of failure.

Keywords: non-linear analysis, safety formats, fracture mechanics.

1 Introduction

The new *fib* Model Code 2010 [1] developed within the international scientific community represents the state-of-the art for design of concrete structures. It reveals trends and ideas for future code development while it is an operational code, useful for practical design. One of the new features is the introduction of a global safety format proposed for verification of resistance assisted by non-linear analysis. This opens possibilities for numerical simulation based on nonlinear analysis to be used as tool in design process. Such innovations observed in other industrial branches are now spreading also to concrete industry.

In a standard design process the load actions are determined for chosen critical cross sections by elastic structural analysis. They represent a possible distribution of internal forces satisfying equilibrium condition, but do not reflect a force redistribution due to nonlinear effects. The safety verification is made in local points whereas a global safety of the whole structure is not evaluated. This deficiency can be removed by applying non-linear analysis.

A non-linear structural analysis based on realistic constitutive relations makes possible a simulation of a real structural behavior. It reflects an integral response, where all local points (sections) interact and therefore it requires an adequate approach for safety verification. The non-linear analysis offers a verification of global resistance and requires a safety format for global resistance.

Several methods for verification of limit states using non-linear analysis are presented: full probabilistic method, method ECOV proposed by the author, global safety factor according to EN1992-2 and partial safety factors. These methods are based on a common probabilistic safety concept for verification of limit states. They differ in the level of implementation of the probability methods. The methods are compared on several examples of reinforced concrete structures ranging from ductile to brittle modes of failure.

2 Global safety formats

2.1 Global design condition

For the verification of resistance the design condition can be approximated by the inequality where the extreme values of actions and resistance are decoupled as follows:

$$F_d < R_d \tag{1}$$

It this F_d is design action and R_d is design resistance and both these entities cover safety margins. The safety of loading and resistance are treated separately, which is a certain approximation as compared to a general probabilistic approach. In design practise based on the partial safety factors this simplification is accepted. $F_d = F(S, \gamma_G, \gamma_Q, \gamma_P, ..)$, where the representative load S is factorized by partial safety factors $\gamma_G, \gamma_Q, \gamma_P, ..$ for permanent load, live load, pre-stressing, etc.

In nonlinear analysis, R_d represents the global resistance in terms of forces corresponding to actions (live load, horizontal load, etc.). Note, that in partial safety factor method we assume a failure probabilities of separate materials, but do not evaluate the failure probability on the structural level. Unlike in sectional design, the global resistance reflects an integral response of the whole structure, in which all material points (or cross sections) interact. The safety margin can be expressed by the safety factor:

$$R_d = \frac{R_m}{\gamma_R} \tag{2}$$

where R_m is the mean resistance. (This is sometimes referred to as *nominal* resistance.) The global safety factor γ_R covers all uncertainties and can be related to the coefficient of variation of resistance V_R (assuming a log-normal distribution, according to Eurocode 2) as

$$\gamma_R = \exp(\alpha_R \beta V_R) \tag{3}$$

where α_R is the sensitivity factor for resistance and β is the reliability index. It is recognized that variability included in V_R depends on uncertainties due to various sources: material properties, geometry and resistance model. They can be treated as random effects and analyzed by probabilistic methods. Due to available statistical data the probabilistic treatment of materials and geometry can be done in a rational way. However, a random treatment of model uncertainties is more difficult, because of limited data. A simplified formulation was proposed in MC2010, where in denominator of the right hand side in Eq.(2) is a product of two factors $\gamma_R = \gamma_m \gamma_{Rd}$. (It follows from determination of partial safety factors in MC2010, Sect.4.5.2.2.3) . The first factor γ_m is related to material uncertainty and can be established by a probabilistic analysis. The second factor γ_{Rd} is related to model and geometrical uncertainties and recommended values are in the range 1.05-1.1. (as suggested by Eurocode 2-2.)

Recent investigation by Schlune et.al. [7] found such values unsafe and proposed a more general method in which the overall coefficient of resistance variation can be determined as

$$V_R = \sqrt{V_G^{\,2} + V_m^{\,2} + V_{Rd}^{\,2}} \qquad (4)$$

Where variability due to specific sources are identified: V_G - geometry, V_m - material strength, V_{Rd} - model. This approach allows to include all uncertainties in more rational way. Based on a survey of various blind bench mark studies Schlune concluded that model uncertainties of nonlinear analysis are much higher than in standard design based on engineering formulas and are strongly dependent on modes of failure. Reported coefficients of variation due to model uncertainty for bending failure in range 5-30%, for shear 15-64%. Schlune concluded that due to the lack of data, the choice of the model uncertainty often depends on engineering judgment and can be subjective. However, these conclusions do not recognize the effect of model validation, which can decrease model uncertainties. Further research is needed to recommend appropriate values of the model uncertainty for numerical simulations.

The assessment of the safety according to Eq.(1) can be done by various methods, ranging from a full probabilistic analysis to the partial factor method, which differ in the level of approximations involved. They will be briefly described in the next sections.

2.2 Full probabilistic analysis

The probabilistic analysis is the most rational tool for the safety assessment of structures. It can be further refined by introducing non-linear structural analysis as a limit state function. The numerical simulation resembles a real testing of structures by considering a representattive group of samples, which can be statistically analyzed for assessment of safety. We shall only briefly outline an approach implemented in the software tool SARA [3]. More about the probabilistic analysis can be found in [6].

In numerical simulations the probabilistic analysis of resistance can be performed by LHS method, in which the material input parameters are varied in a systematic way. The resulting array of resistance values is approximated by a distribution function of global resistance and describes the random variation of resistance. Finally, for a required reliability index β, or probability of failure P_f, the value of design resistance R_d shall be calculated.

The probabilistic analysis based on numerical simulation with random sampling can be briefly described as follows:

Formulation of a numerical model based on non-linear finite element method. Such a model describes the resistance function and can perform deterministic analysis of resistance for a given set of input variables.

Randomization of input variables (material properties, dimensions, boundary conditions, etc.). This can also include some effects, which are not in the action function (for example pre-stressing, dead load etc.). Random material properties are defined by a random distribution type and its parameters (mean, standard deviation, etc.). They describe the uncertainties due to variation of resistance properties. The randomization can be done by two methods: (1) Random variables, where the parameter is constant within a sample (structure), but changes between samples. (2) Random fields, where the parameter is randomly variable within a sample. A correlation of random variables should be considered appropriately.

Probabilistic analysis of resistance. This can be performed by the numerical method of Monte Carlo-type of sampling, such as the LHS sampling method. Results of this analysis provide random parameters of resistance, such as mean, standard deviation, etc. and the type of distribution function for resistance (PDF).

Evaluation of design resistance based on the reliability index β or probability of failure. In this a design point is found by extrapolation of point around central region based on PDF.

The advantage of a full probabilistic analysis is that it is independent of a failure mode. A potentially higher safety margins of some failure modes, such as for example shear failure, is automatically included in higher sensitivity of numerical resistance to a brittle failure. A disadvantage of this approach is in the fact that the target value of design resistance is located in the tail of probability distribution function (PDF), which is determined by the best fit from the sampling. The design value is obtained by extrapolation and strongly depends on the choice of PDF. On the other hand the approach is numerically robust, computationally efficient and feasible for practical application.

However, due to its computational demands a full probabilistic analysis is justified in special cases, where consequences of failure justify the effort.

2.3 ECOV method – estimate of coefficient of variation

A simplified probabilistic analysis was proposed by the author [4] in which the random variation of resistance is estimated using only two samples. It is based on the idea, that the random distribution of resistance, which is described by the coefficient of variation V_R, can be estimated from mean R_m and characteristic values R_k of resistance. The underlying assumption is that random distribution of resistance is according to a lognormal distribution, which is typical for structural resistance. In this case, it is possible to express the coefficient of variation as:

$$V_R = \frac{1}{1.65} \ln\left(\frac{R_m}{R_k}\right) \tag{5}$$

The global safety factor γ_R of resistance is then estimated by Eq.(3) using the typical values $\beta = 3.8$ (50 years) and $\alpha_R = 0.8$ (which corresponds to the failure probability P_f

=0.001). The global resistance factor can be directly related to the estimated coefficient of variation V_R as $\gamma_R \cong \exp(3.04\,V_R)$ and the design resistance is obtained from Eq.(2).

The keystone in this method is the determination of the mean and characteristic values of the resistance: R_m, R_k. It is proposed to estimate them by two separate nonlinear analyses using mean and characteristic values of input material parameters, respectively.

The method is general and the safety described by the reliability index β can be changed if required. Also the distribution function PDF can be changed if justified. It reflects all types of failure. The sensitivity to random variation of all material parameters is automatically included. Thus, there is no need of special modifications of concrete properties in order to compensate for greater random variation of certain properties as in the next method EN 1992-2.

A similar and refined method with more samples was proposed by Schlune et al.[6]

2.4 Method based on EN1992-2

Eurocode 2 for bridges introduced a concept for global safety verification based on nonlinear analysis. Design resistance is calculated from

$$R_d = \frac{R_f}{\gamma_R}, \quad R_f = R(f_{ym}, f_{cf} \ldots) \tag{6}$$

Where f_{ym}, f_{cf} are mean values of material parameters of steel reinforcement and concrete, $f_{ym} = 1.1\, f_{yk}$ and $f_{cf} = 0.85\, f_{ck}$. The global factor of resistance shall be $\gamma_R = 1,27$. Resistance value R_f is not a mean value if the concrete fails. The concrete strength f_{cf} is reduced to account for higher variability of concrete property.

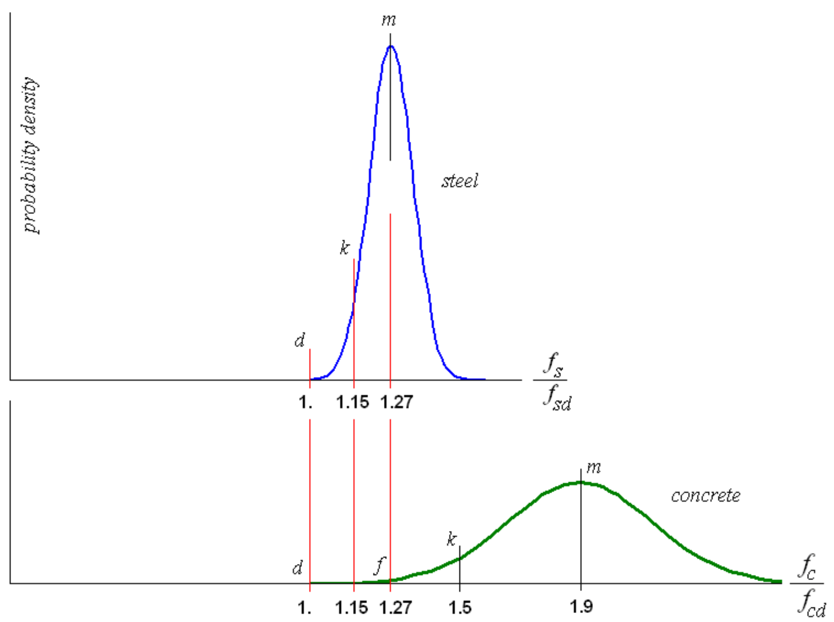

Fig. 1 Probabilistic definition of mean-*m*, characteristic-*k* and design-*d* values for steel and concrete failure, *f*-reduced concrete strength.

A justification for the introduction of concrete strength parameter f_{cf} is based on the idea of reflecting the safety of partial safety factors in the newly introduced global safety factor. The concept illustrated in **Error! Reference source not found.**, where probability density functions for both materials are compared. The strength parameters on horizontal axis are nominal with respect to design values. It is assumed that design values for concrete and steel correspond to the same probability. They are located at point 1. on the horizontal axis. In steel the design value $f_{yd} = f_{yk} / \gamma_s$ is derived from the characteristic strength f_{yk} with the use of partial safety factor $\gamma_s = 1.15$, the mean value is assumed to be $f_{ym} = 1.1 f_{yk}$, which leads to a safety factor 1.27 with respect to mean.

In concrete the design value $f_{cd} = f_{ck} / \gamma_c$, is derived from the characteristic strength f_{ck} with the use of partial safety factor $\gamma_c = 1.5$. We introduce a new parameter for concrete f_{cf}, which correspond to the safety factor of steel for men, and is located at the value 1.27 on the horizontal axis in **Error! Reference source not found.**.

$$f_{cf} = \gamma_R \ f_{cd} = \gamma_s 1.1 \frac{f_{ck}}{\gamma_c} \cong 0.85 f_{ck} \qquad (6)$$

It should be noted that the value of strength f_{cf} does not represent a mean value. Instead, it is a value corresponding to a lower probability than characteristic value and includes the additional safety required for concrete as compared to steel. The subject is also treated by Bertagnoli et al. [8].

The advantage of the above method is, that it covers both models of failure, due to steel and concrete, without necessity of a prior knowledge of failure mode. For concrete the Eurocode 2 allows only compressive type of failure and excludes failure types relying of tension. This, of course, prevents a wide range of applications, such as shear, or pull-out of fastenings. The study presented in [2] extends its applications also to brittle modes of failure.

2.5 Partial safety factors (PSF)

The method of partial safety factors, which is used in most design codes can be directly applied to global analysis in order to obtain the design resistance $R_d = R(f_d)$. In this, the design values of material parameters $f_d = f_k / \gamma_M$ are used for analysis input (f_k are characteristic values and γ_M partial safety factors of materials).

It can be argued, that design values represent extremely low material properties, which do not represent a real material behavior and thus can lead to distorted failure modes. On the other hand, this method addresses directly the target design value and thus no extrapolation is involved. However, the probability of global resistance is not evaluated and therefore not known.

2.6 Comparison of concepts

The methods outlined above offer an estimation of design resistance with various levels of approximations. The full probabilistic analysis is regarded here as the most rational as suggested by the Joint Committee for Structural safety. The other methods are approximations based on simplifying assumptions, which allow estimation of resistance design values. Brief summary of the methods is show in Table 1. and the probabilistic concept of these formats is illustrated in **Error! Reference source not found.**.

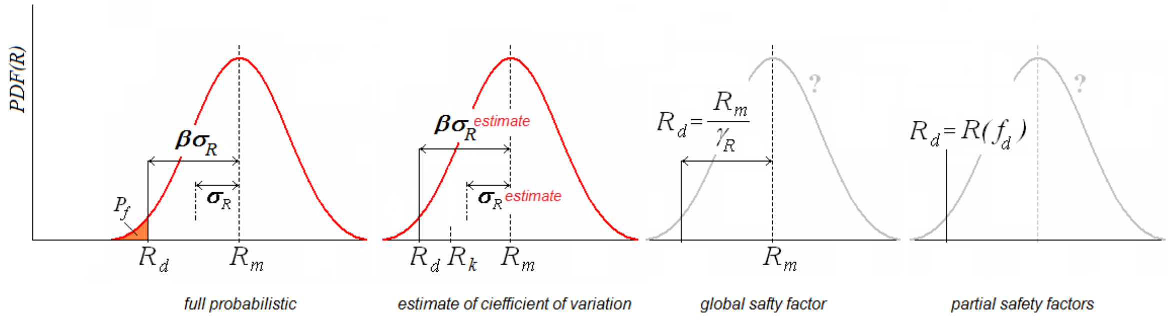

Fig. 2 Probabilistic concept of global safety formats.

Table 1 Summary of methods for verification of global safety

Method	Material parameters required	Required number of resistance calculations	Approximation target
(1) Probabilistic (LHS sampling)	Probability distribution	Depends on number of samples (8-30)	exact
(2) ECOV	characteristic, mean	2	variability of resistance
(3) EN1992-2	characteristic	1	mean
(4) PSF	design	1	design

It has been observed that the safety of resistance depends not only the variation of basic material parameters, but also to the mode of failure. In other words, for the same concrete material, structures with different type of failure can have different variability of resistance. In this respect the most rational approach is by the full probabilistic format (1), in which the random distribution of resistance is determined and the design value of resistance is chosen for a required probability of failure (and reliability index β).

The other three methods (2,3,4 in Table 1) can be regarded as approximate from the probabilistic point of view. The concept of the Method (2) is very close to a probabilistic format, since it works with the variance of resistance and calculations with mean and characteristic parameters are relatively robust. Method (3) is using a unique global safety factor. Assessment of resistance near mean is relatively robust and an effect of concrete variability is included in the reduced concrete strength. Method (4) by partial safety factors, offers a direct estimate of design value without a need of estimating global safety margin. In conclusion each method has its merit and, as will be shown later none seems to be superior to the others.

The author has initiated investigations with the aim to compare the various safety formats [2],[4]. The study comprised of a wide range of structures including: simple beam, laboratory test of shear wall, laboratory test of a deep beam, in-situ test of a real structure bridge and a design case of a bridge pier, SFRC concrete. A variety of failure modes covered ductile bending mode, brittle shear modes and a concrete compression mode. Details of this investigation can be found in [2]. A summary of results is shown in Table 2. Three approximate methods, namely the partial safety factors (PSF), method based on estimate of coefficient or variation of resistance (ECOV) and method according to EN 1992-2 are evaluated. The table shows the ratio of resistances R_d found by approximate methods to the full probabilistic analysis (which is considered as most exact for this purpose). It is noted that the study does not reflect the model uncertainty in a consistent way. The methods PSF and EN1992-2 include the model uncertainty as given by Eurocode, while the ECOV and full probabilistic analysis it is not considered in order to simplify the comparison. This can explain the average results of ECOV method being slightly higher than the other two methods.

Table 2 Summary of methods for verification of global safety.

	$R_d / R_d^{prob.}$		
	PSF	**ECOV**	**EN 1992-2**
Example 1 Bending	**1.04**	**1.04**	**0.99**
Example 2 deep beam	**1.02**	**1.04**	**1.0**
Example 3 bridge pier	**0.98**	**1.04**	**0.96**
Example 4 bridge frame	**0.99**	**0.96**	**0.92**
Example 5 shear beam Y0	**1.03**	**0.98**	**1.02**
Example 6 shear beam Y4	**0.81**	**1.04**	**0.82**
average	**0.98**	**1.01**	**0.95**

The study confirmed feasibility of the approximate methods for the safety assessment. The method ECOV is preferred since it relates the safety to the resistance random variation and is considered more rational as compared to EN1992-2 method.

Multiple failure modes, which are typical features of reinforced concrete structures are inherently included in the numerical models and thus they are reflected in results of analysis and resistance variability. Therefore, the approximate methods of safety verification are generally applicable in design. In significant cases, if justified by failure consequences, a full probabilistic analysis should be applied.

3 Application

For illustration an application of design verification by nonlinear analysis will be shown. The example is a large beam tested in laboratory by Yoshida and Collins [8]. The size of the beam is large and exceeds usual beam dimensions (span=12m, depth=2m) and has no vertical reinforcement. The shear failure is apparently influenced by its large size and is very brittle. The failure mode was well captured by the numerical simulation as illustrated in Fig. 3. Comparison of resistances obtained by various safety formats is shown in Fig. 4Furthermore it shows the values of design resistance by codes EN1992-1 and ACI 318.

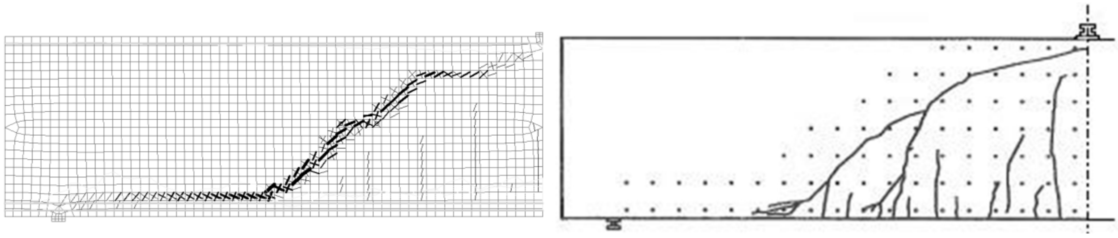

Fig. 3 Numerical and experimental crack pattern.

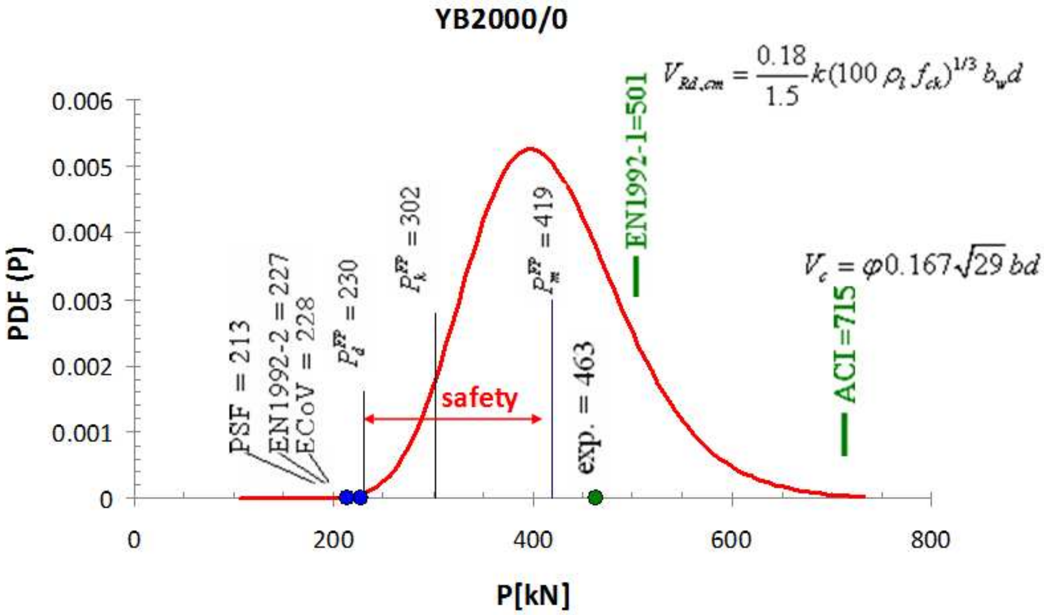

Fig. 4 Design resistance of large beam by Yoshida [8] according to various safety formats and codes.

This case had shown two remarkable features of numerical simulation. First, a refined constitutive modeling based on fracture mechanics can capture the size effect of brittle shear failure and provides a more safe model of resistance. Second, the global safety formats offer consistent safety margins for the design verification.

4 Closing remarks

Model Code 2010 introduced a verification assisted by numerical simulation as one of the design methods and a global safety format. The of application is extended beyond the scope of engineering methods based on elastic distribution of internal forces in cross sections into nonlinear analysis. Due to its general approach it overcomes the limits of standard design based on beams and columns. On the other hand it introduces potentially higher model uncertainties. Therefore the model validation becomes an important requirement for its application in engineering practice.

The *fib* Model Code 2010 outlines the framework of limit state verification by numerical simulations and introduces the global safety formats suggested for this purpose.

Further research is needed in order to improve the guide for validation of numerical models and for the classification of model uncertainties.

Acknowledgement

The paper is based in part on the research project by the Czech Science Foundation no. P105/12/2051. The financial support is greatly appreciated.

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