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On the uniqueness of numerical solutions of shear failure of deep concrete beams: comparison of smeared and discrete crack approaches

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ABSTRACT: Numerical modeling of shear failure of reinforced concrete beams with or without shear reinforcement still remains a challenging task even after several decades of active research. The paper concentrates on the shear failure analysis of large beams without and with shear reinforcement. As an example, it uses the experiments performed by Collins and Yoshida (2006). The paper discusses the main issues affecting the reliability of shear strength predictions and will evaluate the effectiveness of the discrete and smeared crack approaches in resolving the numerical problems in shear failure modeling of strain softening materials.

1 INTRODUCTION

Numerical models for brittle materials such as concrete were for the first time introduced already in the 70's by landmark works of Ngo & Scordelis (1967), Rashid (1968) and Cervenka V. & Gerstle (1971). Many material models for concrete and reinforced concrete were developed in 70's, 80's and 90's such as for instance the models by Suidan & Schnobrich (1973), Lin & Scordelis (1975), De Borst (1986), Rots (1989), Pramono & Willam (1989), Etse (1992) or Lee & Fenves (1998). These models were based on the finite element method. A concrete material model was formulated as a special constitutive model that is used at each integration point for the evaluation of internal forces. It was soon realized that material models with strain softening, if not formulated properly, exhibit severe mesh dependency (De Borst & Rots 1989), and tend to zero energy dissipation if the element size is reduced (Bažant 1976).

This was attributed to the local nature of the constitutive material description, which results in the loss of hyperbolicity of the governing differential equation in the softening region (Belytschko et al 1986). This deficiency means that mathematically a solution can be found, but its uniqueness cannot be guaranteed. In numerical analysis this results in mesh sensitivity and/or numerical instabilities such as convergence problems.

The crack band approach was proposed by Bažant and Oh (1983) to remedy the convergence towards zero energy dissipation. It was shown by Červenka V. (1995) that proper formulation of the crack band size can severely reduce also the mesh bias of these smeared crack approaches.

A more rigorous solution of the ill-posed nature of the strain softening problem is the introduction of higher-order continuum models: such as non-local damage model by Bažant & Pijaudier-Cabot (1987), gradient plasticity model by de Borst & Muhlhaus (1992) or gradient damage model by de Borst et. al. (1996). The non-local models introduce additional material parameters related to an internal material length scale, which is however difficult to derive from existing material tests. Currently these models are mathematically rigorous, but appear to be too fundamental for practical applications.

Another solution for the strain softening problem is the discrete crack model, where the discontinuities arising from strain localization are directly included into the numerical model. This model was first introduced with automatic remeshing and crack propagation by Saouma & Ingraffea (1981). In the classical form of the discrete crack approach a crack is simulated as a cohesive interface, which is inserted into the finite element model whenever a certain criterion for crack initiation or propagation is satisfied. This means that whenever a crack is initiated or an existing crack propagates remeshing is necessary.

In recent years, a whole new class of methods has emerged based on enhanced finite element formulations or various mesh free methods. A comprehensive treatise on these approaches is for instance provided in de Borst et al. (2003) or Jirásek (2003). The mesh free method was proposed by Belytschko et al. (1994). This method has a strong potential for solving crack propagation problems, but it contains still many open issues such as large computational demand, difficulties in 3D implementation and the need to use a background mesh for the numerical integration.

The enhanced finite element formulations such as EED (elements with embedded discontinuities) or X-FEM (extended finite elements) are nicely classified by Jirásek (2003). These models attempt to bridge the gap between the discrete and smeared models by enhancing the finite element formulation to better capture the discontinuity arising from the strain localization. Their basic idea is however very close to the classical discrete approach, where the "traditional" remeshing is replaced by enhancing the finite element formulation in the elements where cracking occurs. Similarly to the classical discrete crack approach it is often necessary to trace the crack propagation through the model, guarantee its continuity across element boundaries, solve crack branching, crack intersection etc.

The paper discusses the main issues affecting the reliability of shear strength predictions. Second objective is to evaluate the effectiveness of the discrete approach in resolving the ill-posed nature of the underlying mathematical problem; since it is often believed that introduction of discontinuities into the numerical formulation can resolve this issue.

The reliable prediction of shear strength in real structures requires a rather complex material formulation, which should capture at least the most important features of concrete behavior, such as: compressive crushing, compressive softening, shear response of cracked concrete and reinforcement yielding. The paper evaluates the importance of these factors on the reliability of shear strength predictions.

2 FRACTURE-PLASTIC MATERIAL MODEL

The smeared crack analyses presented in this paper were performed with program ATENA (Červenka et al. 2009) using the combined fracture-plastic model of Červenka & Pappanikolaou (2008).

The material model formulation assumes small strains, and is based on the strain decomposition into elastic (ε_{ij}^{e}), plastic (ε_{ij}^{p}) and fracture (ε_{ij}^{f}) components. The stress development can be then described by the following rate equations describing the progressive degradation (concrete cracking) and plastic yielding (concrete crushing):

$$\dot{\sigma}_{ij} = D_{ijkl} \cdot (\dot{\varepsilon}_{kl} - \dot{\varepsilon}_{kl}^{p} - \dot{\varepsilon}_{kl}^{f}) \quad (1)$$

The constitutive equations of the both models can be summarized as follows:

Flow rule governs the evolution of plastic and fracturing strains:

$$\text{Plastic model: } \dot{\varepsilon}_{ij}^{p} = \dot{\lambda}^{p} \cdot m_{ij}^{p}, \; m_{ij}^{p} = \frac{\partial g^{p}}{\partial \sigma_{ij}} \quad (2)$$

$$\text{Fracture model: } \dot{\varepsilon}_{ij}^{f} = \dot{\lambda}^{f} \cdot m_{ij}^{f}, \; m_{ij}^{f} = \frac{\partial g^{f}}{\partial \sigma_{ij}} \quad (3)$$

Where $\dot{\lambda}^{p}$ is the plastic multiplier rate and g^{p} is the plastic potential function. Following the unified theory of elastic degradation of Carol et al. (1994) it is possible to define analogous quantities for the fracturing model, i.e. $\dot{\lambda}^{f}$ is the inelastic fracturing multiplier respectively and g^{f} is the potential defining the direction of inelastic fracturing strains in the fracturing model. The consistency conditions can be than used to evaluate the change of the plastic and fracturing multipliers.

$$\dot{f}^{p} = n_{ij}^{p} \cdot \dot{\sigma}_{ij} + H^{p} \cdot \dot{\lambda}^{p} = 0, \; n_{ij}^{p} = \frac{\partial f^{p}}{\partial \sigma_{ij}} \quad (4)$$

$$\dot{f}^{f} = n_{ij}^{f} \cdot \dot{\sigma}_{ij} + H^{f} \cdot \dot{\lambda}^{f} = 0, \; n_{ij}^{f} = \frac{\partial f^{f}}{\partial \sigma_{ij}} \quad (5)$$

H^{p} and H^{f} is hardening modulus for plastic model and fracturing model respectively. This represents a system of two equations for the two unknown multiplier rates $\dot{\lambda}^{p}$ and $\dot{\lambda}^{f}$, and is analogous to the problem of multi-surface plasticity (Simo et al. 1988). The details of the model implementation can be found in Červenka et al. 1998 and Červenka & Pappanikolaou (2008). The model is using Rankine criterion for tensile fracture with exponential softening of Hordijk (1991) (see Figure 1).

The compressive behavior is modeled by the plasticity model, which is using the three parameter surface of Menentrey & Willam (1995) (see Figure 2) and hardening softening is defined according to the laws described in Figure 3 where $\varepsilon_{eq}{}^{p}$ is the equivalent plastic strain. The softening in tension and compression is adjusted using a crack band approach of Bažant & Oh (1983). The crack band L_t as well as crush band size L_c are adjusted with regard to the crack orientation approach proposed by Červenka V. et al. (1995). This method is described in Figure 4 and in (6).

$$L_t' = \gamma L_t \quad \text{and} \quad L_c' = \gamma L_c$$

$$\gamma = 1 + (\gamma_{max} - 1)\frac{\theta}{45}, \quad \theta \in \langle 0; 45 \rangle \quad (6)$$

The basic idea is to adjust the crack band size depending on the crack orientation with respect to the element edges. This reflects the fact that a crack cannot localize into a single element if the crack direction is not aligned with the element edges.

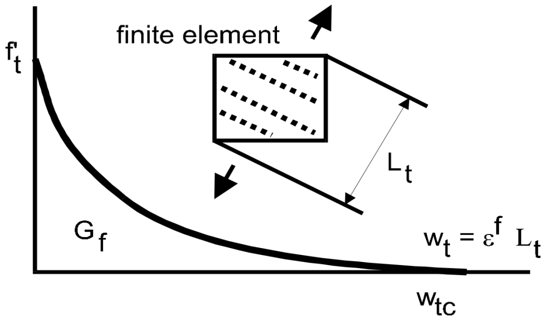

Figure 1. Tensile softening (Hordijk 1991)

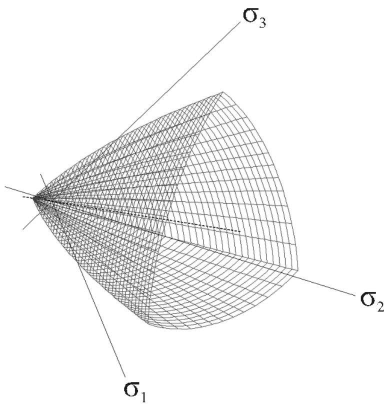

Figure 2. Three paramater criterion for concrete (Menetrey & Willam 1995)

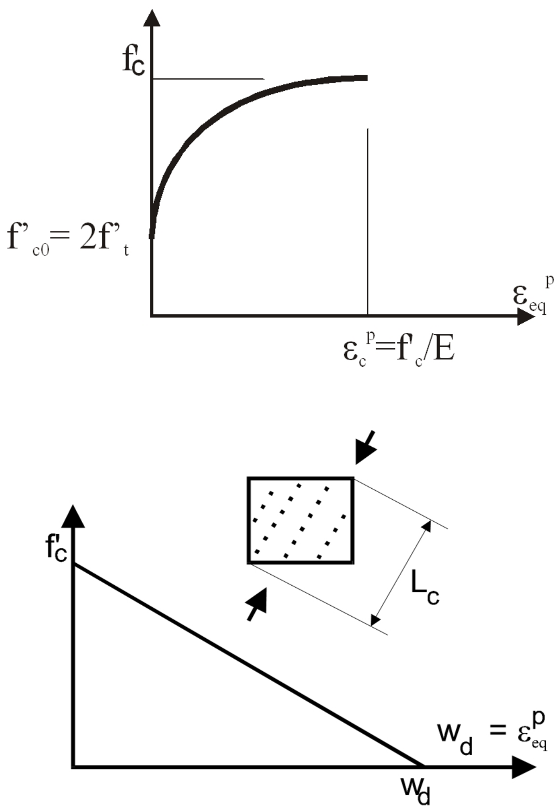

Figure 3. Hardening and softening in compression

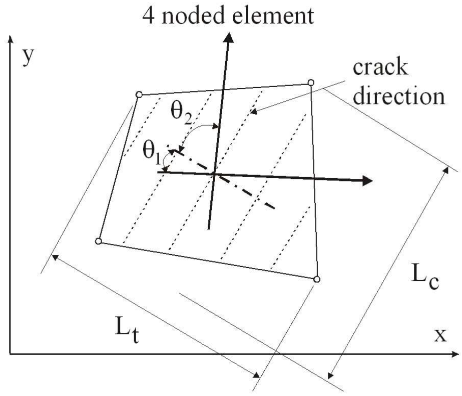

Figure 4. Crack band size adjustment based on crack direction orientation

2.1 *Special features of reinforced concrete*

When it comes to nonlinear analysis of reinforced concrete, i.e. when reinforcement is to be considered, it becomes important to consider additional special issues related to the reinforcement and the composite reinforced concrete material. The most important phenomena are:

a. shear strength and stiffness of cracked concrete, i.e. aggregate interlock
b. compressive strength reduction due to crack opening in perpendicular direction
c. reinforcement yielding
d. tension stiffening
e. dowel action and bending stiffness of the reinforcement
f. bond failure between concrete and reinforcement

In the used constitutive model, the items (a) and (b) are considered according to the modified compression field theory of Collins (Bentz et al. 2006).

In this theory, the compressive strength is reduced using the formula

$$\sigma_c = r_c \, f_c' \tag{7}$$

$$r_c = \frac{1}{0.8 + 170\,\varepsilon_1}, \; r_c^{\lim} \le r_c \le 1.0$$

Where ε_1 is the tensile strain in the crack. In ATENA the largest maximal fracturing strain is used for ε_1 and the compressive strength reduction is limited by $r_c^{\lim}$. In this work $r_c^{\lim} = 0$.

The shear strength of crack concrete is also assumed according to the modified compression field theory MCFT (Bentz et al. 2006) as:

$$\sigma_{ij} \le \frac{0.18\sqrt{f_c'}}{0.31 + \dfrac{24\,w}{a_g + 16}}; \quad i \ne j \tag{9}$$

Where f'_c is the compressive strength in MPa, a_g is the maximum aggregate size in mm and w is the maximum crack width in mm at the given location. The modified compression field theory does not give any formula for the shear stiffness, but it is an important parameter, which significantly affects the reinforced concrete response. In the present formulation the crack shear stiffness K_t^{cr} is calculated directly from crack normal stiffness using a scaling factor s_F.

$$K_t^{cr} = s_F\, K_n^{cr} \tag{10}$$

Where K_n^{cr} comes directly from the tensile softening law in Figure 1 as:

$$K_n^{cr} = {f_t(w_t)}\Big/{w_t} \tag{11}$$

This appears to be a very natural assumption as this makes the shear stiffness dependent on the crack opening displacement.

Reinforcement is modeled using the embedded approach with truss elements, and a multi-linear stress-strain law is used to capture reinforcement yielding.

Tension stiffening can be activated in the present model, but was not used. It was shown by Červenka & Margoldová (1995) that if sufficiently fine mesh is used the tension stiffening effect can be very well captured by an appropriate cracking model.

The dowel action and reinforcement bending stiffness is not considered in the present model. The analyzed beams are only slightly reinforced; therefore these effects cannot play a major role.

Reinforcement bond failure can play an important role in the analyzed problems. A bond modeling was discussed by authors in a separate paper Jendele & Červenka (2006). It was shown that a bond model can strongly improve the results if large finite elements are used in heavily reinforced structures. The problems presented in this paper are only lightly reinforced and the largest element size is 200 mm. It was therefore decided not to use the bond model to limit the number of investigated parameters.

3 DISCRETE CRACK MODEL

The analyses calculated with the discrete crack model in this paper are using a simple approach, where a crack is modeled as a zero thickness interface with Mohr-Coulomb type of criterion with tension cut-off (see Figure 5).

$$\tau \le c - \sigma\phi;\ \sigma \le f_t \tag{12}$$

Where c is cohesion and ϕ is frictional coefficient. The Hordijk's (1991) law is used for tensile softening. The cohesion softening is also modeled by the same law but the displacement values in the softening diagram for cohesion are 10 times increased.

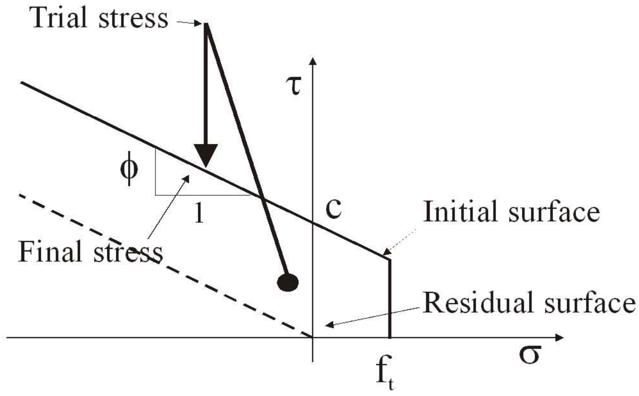

Figure 5. Mohr-Coulomb criterion for discrete crack elements

This approximately corresponds to the assumption that the shear response should be more ductile then the tensile one, and that the mode II fracture energy G_F^{II} is about 10 times larger then the fracture energy for mode I.

$$G_F^{II} \cong 10\, G_F^{I} \tag{13}$$

4 SHEAR FAILURE IN PLAIN CONCRETE

In the first example, the discrete and smeared crack models are compared on a typical shear problem without reinforcement (see Figure 6). This is the well known Iosipescu's shear beam. The geometry corresponds to the SEN beams tested by Schlangen (1993). The tested beams have dimensions 440x100x100 mm. They were cast from concrete with modulus of elasticity E =35 GPa, Poisson's ratio $\nu = 0.15$, tensile strength f_t=2.8 MPa and the specific fracture energy G_F = 70 N/m.

This test setup was originally proposed by Iosipescu (1967) for shear tests of metals. This test was later used by Bažant & Pfeifer (1986) for shear testing of concrete. It was discovered by Ingraffea & Panthaki (1985) that the crack propagation in this

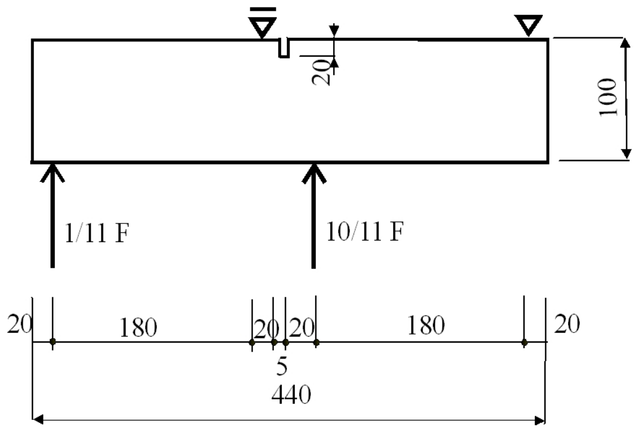

Figure 6. Geometry of the modified Iosipescu's beam (Schlangen 1993), the dimensions are in mm.

kind of test is mainly dominated by mode I, i.e. tensile cracking. Since then it has become a typical test problem for crack propagation analysis, because it is a common believe that smeared crack models cannot predict the behavior correctly and some kind of enhanced formulation is necessary.

The load displacement diagrams are compared in Figure 7. The figure shows a single discrete crack analysis and several smeared crack results. The discrete crack analysis has been performed previously by Červenka J. (1994). The peak load in the discrete crack analysis is captured very well as well as the overall shape of the response. In post-peak the response is slightly lower. This could be probably improved by increasing the shear properties of the cohesive interface model. The crack path was determined (see Figure 8) by the direction of maximal principal stresses at the crack tip. The discrete crack model can nicely capture the curved shape of the crack path. On the contrary, the crack path curvature is even slightly overestimated. This is caused by extending the crack by a certain non-infinitesimal length Δa at each propagation. Because of that the crack extension is overestimated, and the crack needs to curve strongly to return to the correct path.

Figure 1 shows also the results from several smeared crack analyses. All smeared crack analyses showed the crack path depicted in Figure 9, i.e. a more or less straight crack path towards the right side of the bottom loading plate. So the curved crack path is not obtained, but the crack ends at the right side of the loading plate. For instance the smeared crack results reported by Schlangen (1993) are strongly affected by the mesh bias, and an incorrect vertical crack is reported, which ends to left of the bottom plate. This improved behavior of the current model can be attributed to the crack band size formulation (6).

The smeared crack analyses labeled with "Std" indicate analyses using the model described in Section 2, but the special features for reinforced concrete are not activated. The shear factor s_F is set to the low value of 20, which means that the shear stiffness on the crack surface is almost identical to the normal one. The main findings from this study can be summarized as follows:

a. The results confirm that the crack propagation is mainly in mode one.
b. The shear properties of the crack do not influence the results significantly.
c. The cracked area is quite localized so no numerical problems occur in the smeared crack analysis
d. The peak load is predicted correctly by all models.
e. The coarse finite element models show lower peak values, which is quite common situation in the crack band model.

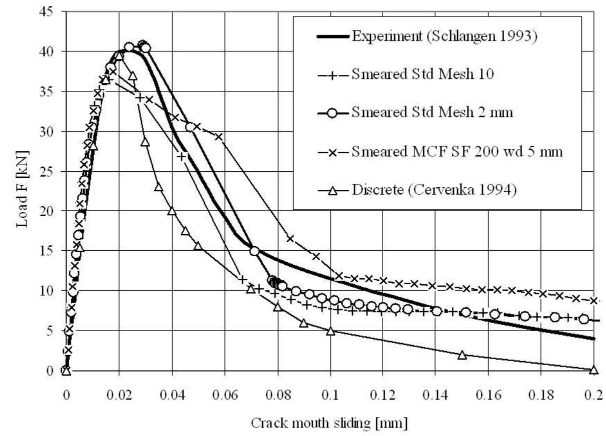

Figure 7. Comparison of load-displacement diagrams for the Iosipescu's beam.

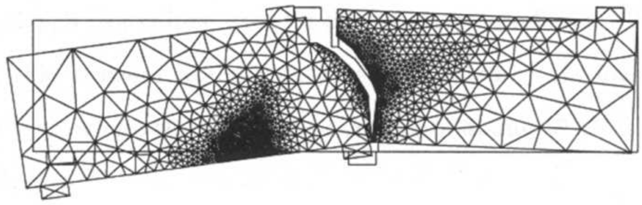
Figure 8. Crack pattern by discrete analysis.

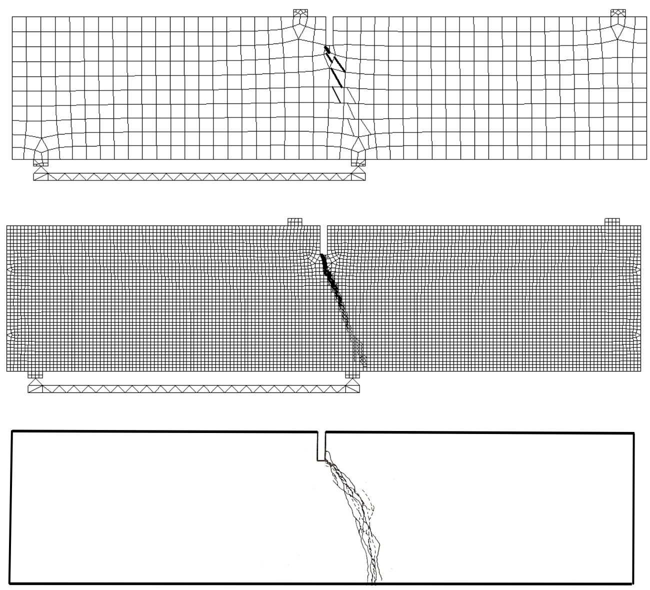
Figure 9. Crack pattern by smeared analysis. (a) coarse mesh 10 mm, (b) fine mesh 2 mm, (c) experiment (Schlangen 1993)

f. The discrete model predicts more accurately the curved shape of the crack path.
g. The crack path predicted by the smeared model is acceptable for practical applications.

5 SHEAR FAILURE IN LARGE RC BEAMS

In the next example large beams tested at the University of Toronto by Collins and Yoshida (2000) were investigated numerically. Two beams from the experimental program of Yoshida are considered:

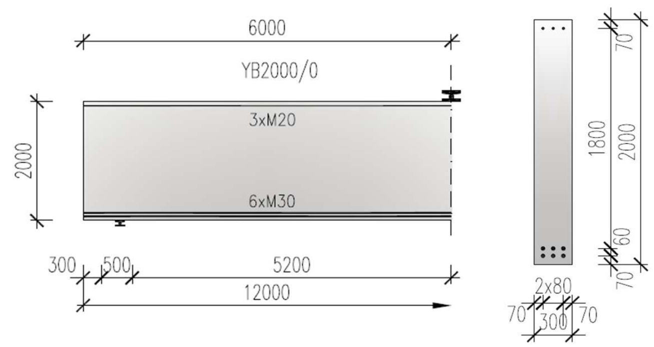

Figure 10. Beam YB2000/0 dimensions and reinforcement.

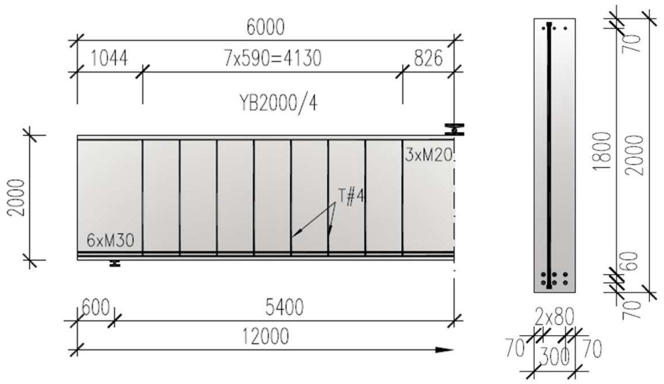

Figure 11. Beam YB2000/4 dimensions and reinforcement.

Beam YB2000/0 with bending reinforcement and no shear reinforcement and beam YB2000/4 with vertical reinforcement by 8 T-headed bars. The beams are schematically depicted in Figure 10 and 11.

The longitudinal reinforcement in both beams is identical. The reinforcing ratio of bottom reinforcement of 6xM30 bars is 0.0074. The ratio of vertical reinforcement of T-headed bars T#4, spacing 0.59m is 0.00071. The beams are only lightly reinforced. The shear span ratio a/d=2.86 indicates a shear critical geometry.

The experimental study Yoshida (2000) offered for concrete property only a compressive strength at the date of testing, which was obtained from cylinder tests. In the tests, slightly different properties were found in two specimens. However, in this study it was decided to use identical concrete properties in both specimens in order to keep the effect of different shear reinforcing not influenced by other parameters. The assumed set of parameters for concrete and reinforcement is shown in Table 1 and 2. The parameters reported in this table are referred to a „Standard" or "Std". In some analysis certain parameters are modified to evaluate their influence on the results.

The finite element analysis was done for a symmetrical half of the beam in plane stress representation. Quadrilateral 4-node isoparametric elements, sizes 50-200 mm, were used for concrete and embedded truss elements for bars. The total load $P = 2V$ acting in the top centre of the beam is considered as the global resistance. Like in experiment, self weight is considered in the analysis but not included in the monitored load P.

Table 1. Concrete material properties of RC beams, this material set is denoted as "Std!, i.e. "Standard".

Concrete property	Value
Elastic modulus E_c [MPa]	34 000
Compressive strength f_c [MPa]	37
Tensile strength f_t [MPa]	2.8
Specific fracture energy G_f [N/m]	80
Poisson ratio μ [-]	0.2
Plastic strain at f_c (peak) ε_{cp} [-]	0.001
Plastic end displacement w_d [mm]	0.5
Shear factor s_F	20
MCFT f_c reduction	none
MCFT aggregate interlock	none

Table 2. Reinforcement properties of RC beams

Steel property	Value
Elastic modulus E_s MPa	200 000
Yield stress f_y MPa	470
Max. stress $f_{s,,max}$ MPa	680
Limit strain ε_{lim}	0.11

5.1 *Discussion on best-fit results*

The material properties denoted as "Std" and listed in Table 1 correspond to a standard material setup. It is approximately identical with the standard EC2 concrete class C30/37. The used set of material parameters can be recognized as mean properties of this concrete class.

$$f_{ctm} = f_{ck} + 8 = 30 + 8 = 38 \approx 37 \text{ MPa} \tag{13}$$

This was the initial set used for the analyses. This set of parameters is very similar to the one used in Section 4. This set of parameters does not include any of the special provisions for reinforced concrete analysis from Section 2.1. It provided very good results for the Iosipescu's beam (see Figure 7) and also for the case of beam YB2000/0 (see Figure 10). The load-displacement diagrams for this beam are compared in Figure 12. However for the beam with stirrups (Figure 11), the peak load was greatly underestimated. These results are reported in Figure 22 under the label "FP-Std". The peak load is underestimated by almost 50%. The input parameters had to be modified in order to obtain a good agreement. This best-fit response is shown in Figure 13, and the adjusted parameters are listed in Table 3.

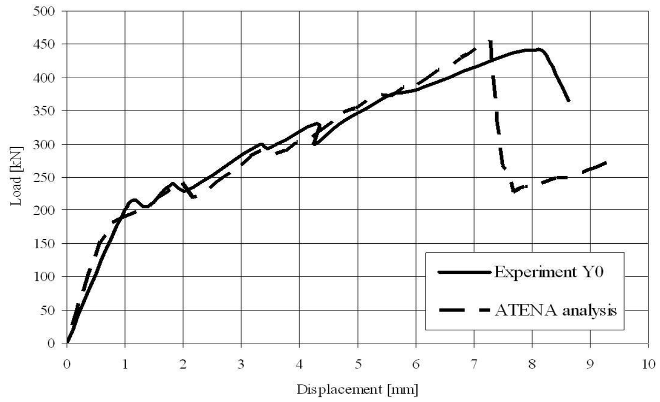

Figure 12. Beam YB200/0 L-D diagram comparison. Analysis is based on „Std“ properties and mesh 200 mm.

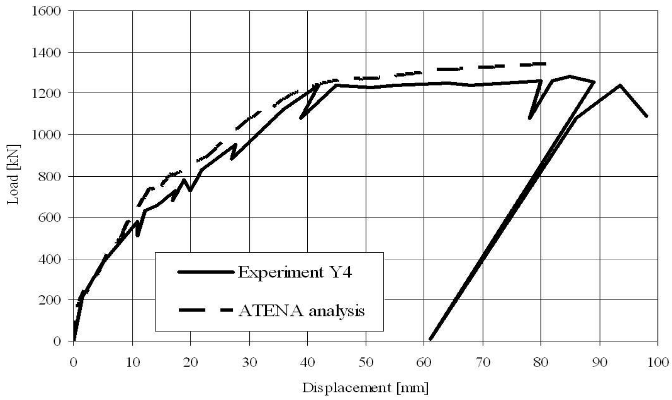

Figure 13. Beam YB2000/4 L-D diagram comparison. Analysis is based on modified „Std“ properties, s_F=300, ε_{cp}=0.002, w_d=50 mm

Table 3. Adjusted parameters for best fit for beam YB2000/4

Concrete property		Value
Plastic strain at f_c (peak):	ε_{cp} [-]	0.002
Plastic end displacement:	w_d [mm]	50
Shear factor:	s_F	300

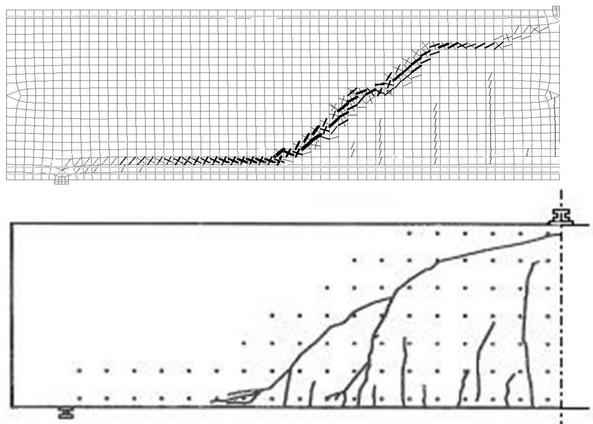
Figure 14. Beam YB2000/0 crack pattern comparison.

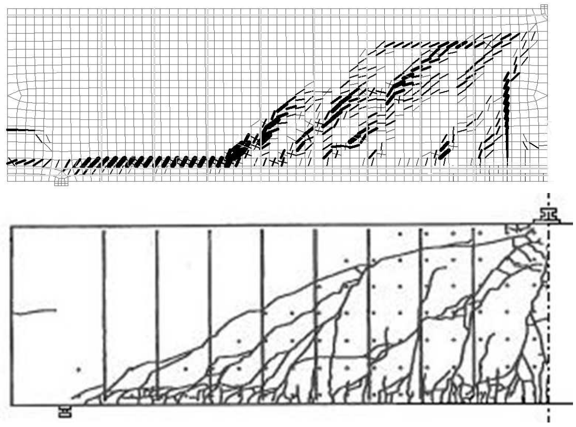
Figure 15. Beam YB2000/4 crack pattern comparison.

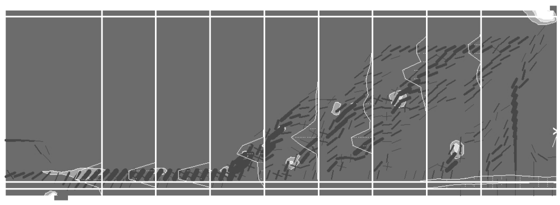
Figure 16. Beam YB2000/4 concrete crushing and yielding of stirrups in numerical analysis.

From the adjusted parameters it is clear that the deficiencies of the initial parameter set were:

- brittle response in compression
- low shear stiffness of the cracked material

The beam YB2000/0 is failing due to a diagonal cracking. The diagonal cracks can fully open and therefore no significant shear stress can be transferred across the cracks. This failure pattern is nicely documented in Figure 14, which also shows a good agreement between the calculated and observed crack patterns.

This should be contrasted by the behavior of beam YB2000/4. This is a beam with shear reinforcement. The reinforcement limits the crack opening so the crack cannot open so much, and significant shear is transferred across each crack. If the shear stiffness is underestimated, a premature failure is calculated. Figure 16 shows the calculated failure mode for this beam. The final failure is due to concrete crushing near the top loading plate and stirrups yielding. Also the bottom bending reinforcement is yielding at this point. However, to obtain a ductile response as in the experiment, it is necessary to increase the ductility of the concrete in compression; otherwise the concrete near the top loading plate fails by a brittle compression failure.

In both examples, it is rather difficult to obtain a stable solution in the post-peak. This can be attributed to the following facts:

- It is a large beam with lot of elastic energy, which needs to be released.

- Large areas of the model are cracked, and there exist multiple similar solutions, which of these cracks should close and which to open.

The second point exactly corresponds to the deficiency of the smeared crack models reported in the

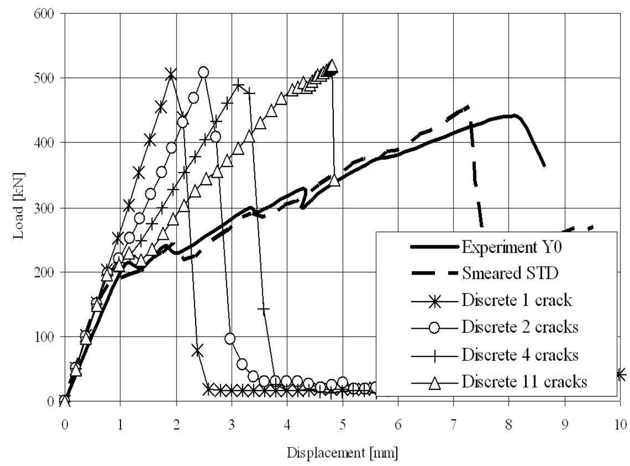

Figure 17. Beam YB2000/0 results including discrete analyses.

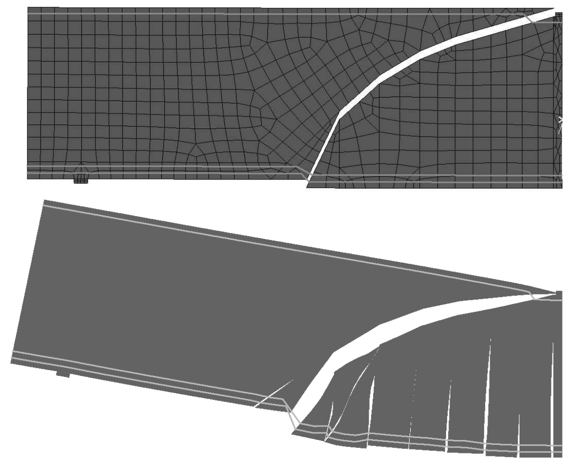

Figure 19. Discrete models for beam YB2000/0 with 1 and 11 discrete cracks

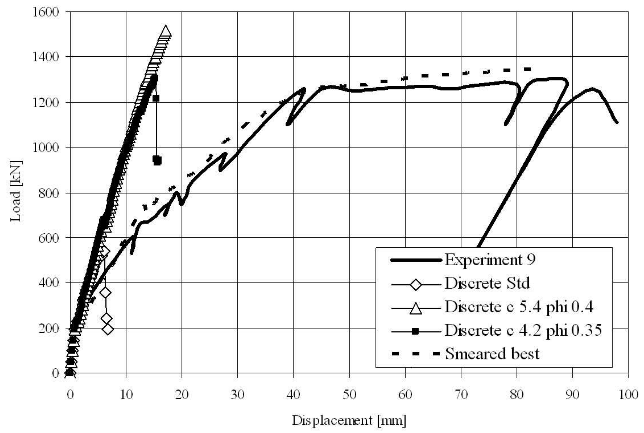

Figure 18. Beam YB2000/4 results including discrete analyses.

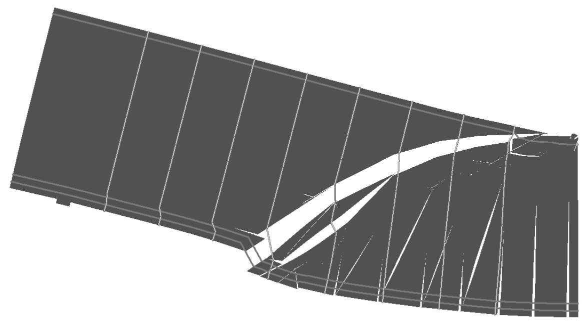

Figure 20. Discrete model for beam YB2000/4 with 22 discrete cracks

introduction to this paper. The mathematical problem of the strain-softening material becomes ill-posed and the uniqueness of the solution is not guaranteed.

5.2 *Discrete crack analyses*

Now, it will be interesting to explore if an application of discrete crack model can help to resolve this issue of non-uniqueness and numerical stability.

Both beams are analyzed using a discrete crack model with cohesive zero thickness elements as described in Section 3. These elements are placed along the expected crack paths. It should be noted that in this study no automatic remeshing and crack propagation is used. It is not necessary since the objective is to verify if the addition of discrete discontinuities into the model can help to resolve the localization problem of the strain softening material. During the localization process some of the initially created cracks need to close while some should open.

The results from the discrete crack analyses are summarized in Figure 17 for the beam YB2000/0, i.e. the beam with no shear reinforcement. In this figure several discrete analysis are shown with different number of inserted cohesive cracks. The number of assumed cracks ranges from 1 to 11. The first crack, which quite naturally comes to our mind, is the diagonal shear crack (Figure 19 top), which corresponds to the final failure mode of this beam as it was shown in the previous Section 5.1 in Figure 14. The diagonal crack is not the first crack that appears in reality. The previous analyses showed that the cracking is first initiated in the middle of the beam as bending cracks that later on spread through the whole bottom part of the beam. When these bending cracks are not included in the discrete model an extremely stiff response is obtained as shown in Figure 17.

In order to correct the pre-peak stiffness, new models were created with multiple bending cracks in the middle of the beam. One such model is shown in Figure 19 (bottom) with altogether 11 discrete cracks. It is interesting to note that as the number of discrete cracks in the model is increasing the stiffness of the pre-peak is improving as well. Figure 17 also shows that once the number of discrete cracks increases to 11 it becomes very difficult to obtain a stable post-peak solution.

Analogical results were obtained for the beam YB2000/4. In this case, a model with cca 22 discrete cracks was used (Figure 20). The load displacement diagram is shown in Figure 18. It is clear that even

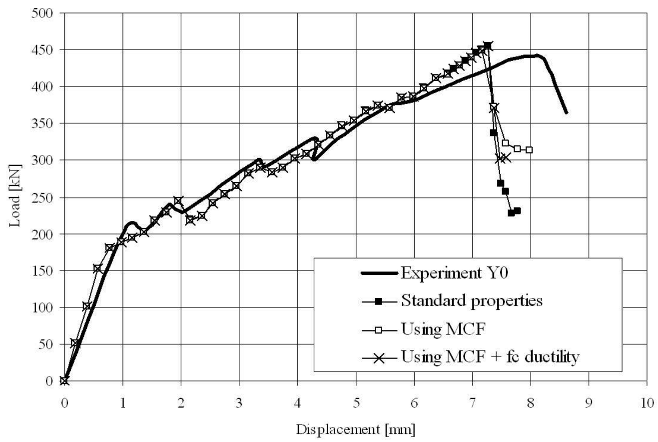

Figure 21. Beam YB200/0 effect MCFT and ductility.

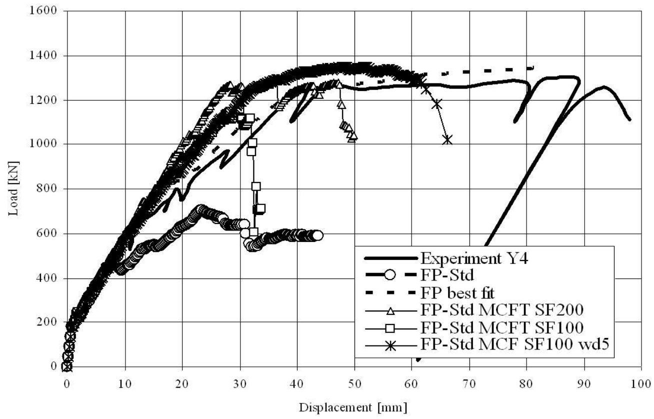

Figure 22. Beam YB2000/4 effect of MCFT and ductility.

22 discrete cracks are not enough to capture the reduction of stiffness due to the diffused crack pattern in the pre-peak regime. The results show that similarly to the smeared model in Figure 13, it is necessary to modify the shear properties of the crack model to obtain at least a correct peak load. Standard discrete parameters underestimated the peak load by more then 50%. It was necessary to increase the cohesion to 4.2 MPa and friction coefficient to 0.35 to obtain good match of the peak load (Standard values were c = 2.8 MPa and $\phi = 0.3$).

It was also very difficult to obtain a stable solution once the peak load was reached. Many of the discrete cracks are opened, many similar solutions exist. For the numerical solver it is difficult to determine which of them should close and which should continue to open and localize the failure.

5.3 *Effect of special reinforced concrete features*

Various special issues related to the constitutive modeling of reinforced concrete were introduced in Section 2.1. It will be interesting to examine their effect on the numerical solution. Some effects were already discussed in Section 5.1 and 5.2 and additional results are shown in Figure 21 and 22.

Figure 21 clearly shows that these special features play only a minor role when no shear reinforcement is present. This is also confirmed by the results of the Iosipescu's beam in Section 4.

Totally different situation is in the case of beam YB2000/4, i.e. the one with shear reinforcement. Although the beam is only lightly reinforced, its strength is determined by reinforcement yielding as shown in Figure 16. The results also show that the shear properties of the crack concrete, i.e. both the shear strength as well as the shear stiffness should be considered properly. In this case the shear stiffness of the cracked concrete was a major factor. The MCFT (Bentz et al. 2006) parameters such as the aggregate interlock and the reduction of compressive strength due to cracking did not play a major role. As already pointed out in Table 3 it was the concrete compressive ductility and shear stiffness of cracked concrete that proved to be critical for good prediction of the beam behavior.

6 CONCLUSIONS

Paper discusses various aspects of numerical predictions of shear strength of plain and reinforced concrete structures. One of the objectives is to verify whether the introduction of strong displacement discontinuities into the numerical solution can be used as a remedy for the known problem of softening materials, i.e. the ill-posed nature of the mathematical solution, which results in a non-unique solution.

In plain concrete the discrete crack model definitely improves the crack path predictions; however a good smeared crack model can provide almost identical results. This is especially true if the randomness and heterogeneity of the concrete material is taken into account. In reality, the crack path will always differ in all tests, so minor deviations from the exact path should be tolerated.

In reinforced concrete, the discrete crack model is applicable only if large number of discontinuities is introduced into the model. This may be difficult to accomplish with the classical form of the model with remeshing, but can be handled by its modern variants such as X-FEM. With increasing number of discontinuities, i.e. cracks, it is apparent that the same problem of solution non-uniqueness will appear. This shows that the enhanced finite element method cannot be used as a remedy to this problem of softening materials. The only proper solution would be a non-local approach or a full dynamic analysis with rate dependent formulation.

The reinforced concrete beam shows that shear properties of the crack concrete are critical for good predictions, although the current level of knowledge is quite limited in this area. The aggregate interlock as well as the f_c reduction proposed by Modified Compression Field theory of Bentz et al. 2006 did not play an important role for the shear strength of the analyzed beams.

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